Mathematically Driven Forms and Digital Tectonic: A formula for realizing the digital

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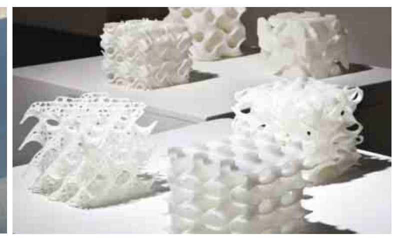


Abstract

Mathematics has been the interest of architects for hundreds of years and has been used in projects ranging from the Denmark Pavilion at Expo 2010 to Gaudi's cathedral. Generative form finding frequently takes the inspiration of the geometric aesthetic found in mathematic forms. Today, the influence of digital computation technology is increasingly evident in architectural form seeking and analysis as they relate to mathematics. The sculptural possibilities of math forms have reconditioned the design process that establishes new modeling and tectonic approaches. This paper focuses on the study of current constraints and new procedures within mathematical approaches to architecture. Furthermore, this paper describes three experimental projects exploring mathematically driven designs and their potential within architectural vocabulary. In these experiments, the designers and students explored the manipulation of a planar surface through algorithmic equations and the molecular make-up of a surface through voxel representation.

1. Introduction

Today, mathematical computation offers new potentials in producing various math formulations that are used to construct 3D forms through control variables. The results of this new exploration are flourishing due to the unlimited possibilities and expanded boundaries of the imagination. Greg Lynn believes that "one exciting and important potential offered by these technologies (computational) lies in the exploration of shapes and forms far too complex to attempt with ruler and compass. Furthermore, design "in the computer" also raises the possibility of removing physical constraints from the modeling process" (Lynn, 2008). These unlimited possibilities of computational forms must still be converted to building "mass, structure, or texture" (Baerlecken, 2008). Designers and architects have to deal with these unconventional artifacts using engineering and manufacturing logic, which focuses on constraints (constructability, materiality, and scale) rather than aesthetics.

2. Constraints of math objects

Over the past several years, designers have showcased the ways to build on a component level that represents the micro scale artifacts of a macro scale environment. By exploring the ways in which the designer can utilize the current fabrication machines, to produce at a macro scale, will inevitably progress the current construction processes and allow for digital fabrication to be an integrated component into the design process. As the architecture field continues to exploit these digital fabrication technologies, the only question that continues to emerge is that of scalability. How can complex math models paired with practical tectonic approaches relate within the architectural scale and current construction technology?

Experimentation in mathematics, 3D algebra and 4D (3D + time), can yield new forms for fabrication and assembling. Digital fabrication has given architects the tools needed to manifest the conceptual ideas into the built environment and populate our habitable landscape with artifacts that were deemed "unbuildable" and only realized in the virtual world. Digitally generated mathematical solutions lend themselves to fused deposition modeling (FDM), CNC milling or laser cutting fabrication pipelines. However, the massive scale of architecture trumps the micro scale of most digital fabrication machines. For instance, 3D printing with FDM is ideal to fabricate small scale sculptural forms, but not practical to manufacture large scale architectural forms. Part of the design challenge, when designing with digital fabrication as a method to materialize the mathematic model, is to be able to realize the conceptual idea with the allowance of the current fabrication tools.

There are several computation techniques that have been widely adopted to prepare digital models for fabrication such as panelization, tessellation, rigging, waffle assembling, and folding. In these methods, a large complex math form, such as a hyperbolic surface, has to be subdivided to allow machines to fabricate components such as ribs and panels and then assemble them as a single structure. However, due to the increasing level of complexity of math forms, especially self intersecting and interlocking form, this subdivision process can be rarely parametrically controlled. Neither the rigging nor waffling methods can successfully subdivide and reconstruct the complex math objects.

3. Alternative approaches

3.1 Slicing

The complexity of an algebra surface or any mathematically generated artifact can be easily manufactured by converting the surface to a thickened shell and then cut uniformly along one axis. The slicing or contouring technique combined with stacking geometries results in a fluidly reconstructed math object which is obtainable due to the extraction of planar 2D chips. Different algorithms can be applied to the math object in order to generate a sequence of enclosed 2D profiles which are often taken at a set distance. Prototypes or scaled models can use material thickness to control the location of the contours by determining the heights needed on a per floor basis (Figure 1). Through this slicing method, a complex form can be easily divided into a large quantity of 2D shapes or contour chips. Each chip can be realized and labeled through digital fabrication with laser cutter or CNC milling machines and then re-assembled to reveal the mathematical form with a set vertical step-over ratio. "As architects increasingly design with complex geometries, using slicing as a method of taking numerous cross sections through a form has proven time and again an effective and compelling technique. As in conventional construction processes, information is translated from one format to another to communicate with the builder – only in this case the builder is a machine." (Iwamoto, 2009).

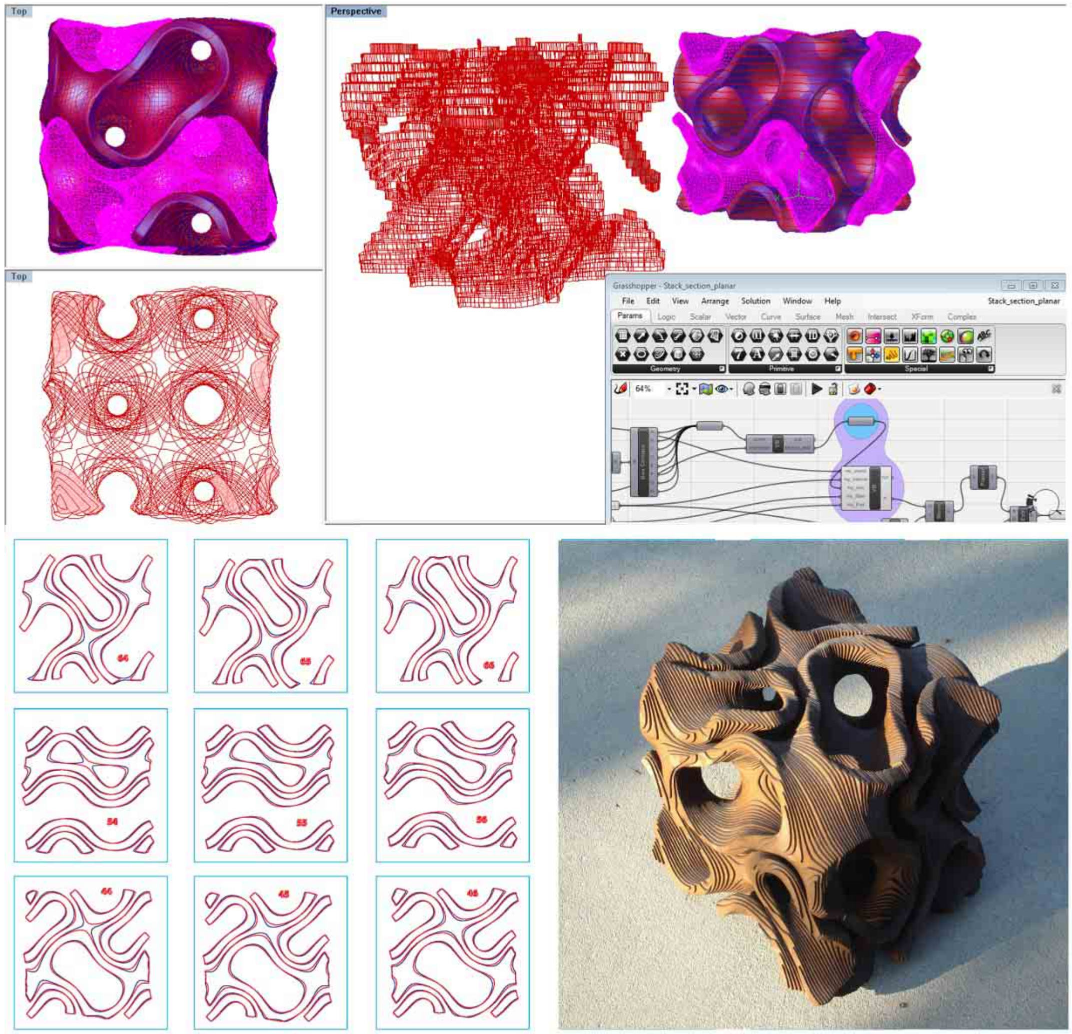
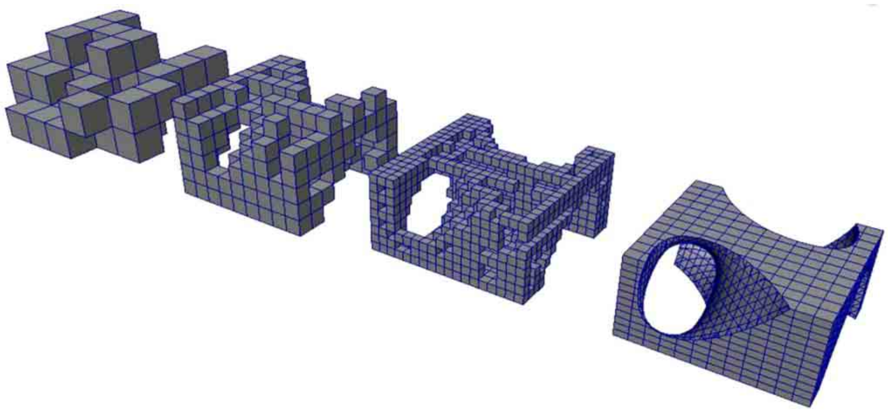


Figure 1: Slicing math objects. Top: parametric control of slicing; Bottom: interlock object for laser cutting

3.2 Spatial Occupancy Modeling

A similar approach to the slicing technique is voxelization through Spatial Occupancy (SO) modeling. Inspired by Magnetic Resonance Imaging (MRI), the Spatial Occupancy modeling was investigated for its capability of representing complex geometries in the molecular level. It is effective to compile a set of contiguous discrete “chunks” or voxels to define a SO model. Then the local surface feature is uniformly represented by small voxels through x, y, z coordinates. Large quantities of voxels are arrayed in space and thus formulate the presented mathematical form. The characteristics of the form are heavily relied on the voxels’ spatial relationship and the internal logic among themselves.

Inspired by in the construction of Great Pyramid, and the Lego house designed by Barnaby Gunning in 2009, the authors articulated this voxelization technique in various programs such as Maya python, Houdini, Grasshopper and Mathmatica before it was introduced to a studio course. The internal parameters, such as voxel's size, local placement, rows, columns, and subdivisions, are subjected to alterations controlled by a graphical interface. By manipulating these parameters the quantity and array density for the voxelization process rebuilds the surface in either a more abstract manner or fluidly mirrors the original input surface (Figure 2). With the authors' approach, any complex math form can be easily subdivided and represented by a newly generated 3D form which is easily associated with conventional construction technique, such as masonry clustering.



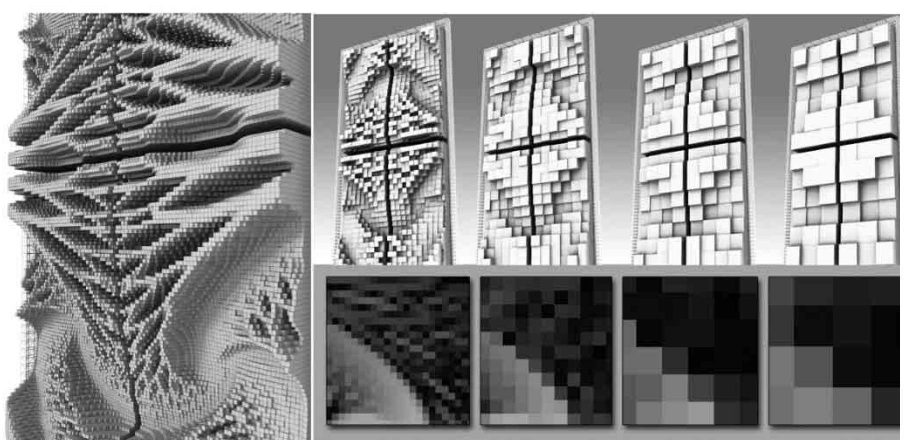


Figure 2: From top to bottom: Voxelization of mathematic form made by authors; skyscraper design by authors.

4. Experimental projects

4.1 Mathmorph

The name "Mathmorph" combines the notion of "mathematics" with the notion of "morphology". This project focused on the study of mathematics as an embedded variability of spatial arrangement with a procedural model. Mathmorph also investigated how to fabricate unconventional math forms in order to explore their potentials of being used as architectural forms. The project used several experimental approaches to facilitated 3D form generation, visualization and fabrication.

First, a series of computer models were generated using computer algorithms and mathematic equations[1]. Secondly, a series of 3D models were generated by importing these computer algorithms and mathematic equations into CAD programs. These computer models were fabricated as physical prototypes by the FDM systems, CNC machine, and laser cut machines. The purpose of this part is bi-fold. It does not only inspire designers to use unconventional math forms in architectural design, which has traditionally been restrained by the difficulties in design and visualization, but also tests the possibility of these unconventional forms in being manufactured as physical prototypes.

A series of abstract building masses designed with the focus on their potential transformative spatial layouts was also explored. The generation of an abstract mathematic form using equations was studied as non-conventional forms which manifested interlocking / intertwining between solid forms and void spaces. We adapted several variables to control the repetition and resolution of these interlocking spaces by an exhaustive combination of several variables values. From a large number of outcomes, only several ideal spatial arrangement solutions were selected by the authors and then used as the genotype for the next operation.

Here, the math form was considered as a solid mass and "sliced" into a multi-story skyscraper. The authors were able to parametrically control the distance between each cut plane, or floor slabs, in order to achieve the aesthetic desired. The results were a series of drawings and models that revealed possible construction methods of birthing complex mathematical forms in the built environment. This computational approach combined two areas of interest, digital form finding and digital fabrication, to produce a formula for realizing the digital.

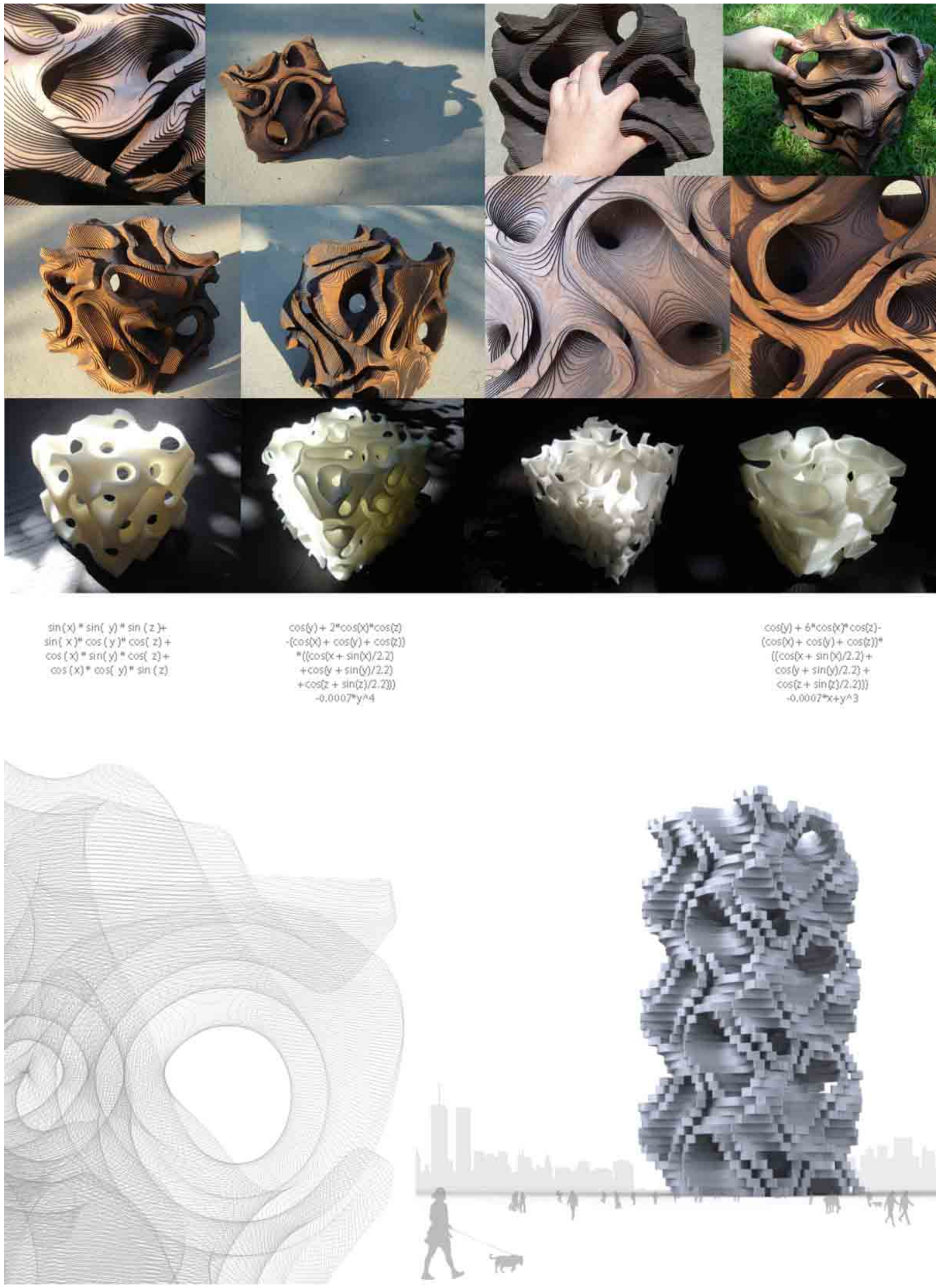


Figure 3: Mathmorph project. FDM fabrication of gyroid surface and laser cutting fabrication of gyroid surface. http://mathmorph.com

4.2 Fractal Imprint

Fractal Imprint explored the voxelization of a math-driven building skin through a mathematical range and division. First, the complexity and resolution of the surface were controlled mathematically by 2D patterns such as fractals. The 2D image was generated using mathematic equations and translating its unique pattern to 3D displacement values. The quantity of pixels of the 2D image controlled the quantity of voxels used to “build up” the 3D panel form. This computational approach encapsulated a unit’s relationship to other units and ultimately altered the building façade. The authors created a high degree of complexity and explored the dynamic possibilities of spatial arrangement with relatively simple input information.

The complexity was easily fabricated by components built into the parametric model that produced the file documentation that is needed to realize the artifact in the physical landscape. In this process, the voxel based mathematic model demonstrated an unlimited potential of form exploration from sets of parameters. The authors selected the desired control parameters to manipulate the quantity of voxels which represented the mathematic form and generated spatial organization on the façade surface (Figure 4).

This project ultimately proved that a complex math model can be optimized by a parametrically controlled voxelization process. The authors investigated the voxel study by milling a voxel surface out of high density foam and vacuum forming craft foam to generate a modular tile that acts as a unit that can be multiplied to generate new patterns. The modular patterns that were generated through this exploration can be viewed as interior acoustical panels or a scaled building façade model. Regardless of its interpretation, the voxel surface is manifested from each voxel’s height value being controlled by its corresponding 2D image’s alpha value. These results lend themselves to being studied at a smaller scale and investigated as an architectural element that could alter the interior architecture of any context.

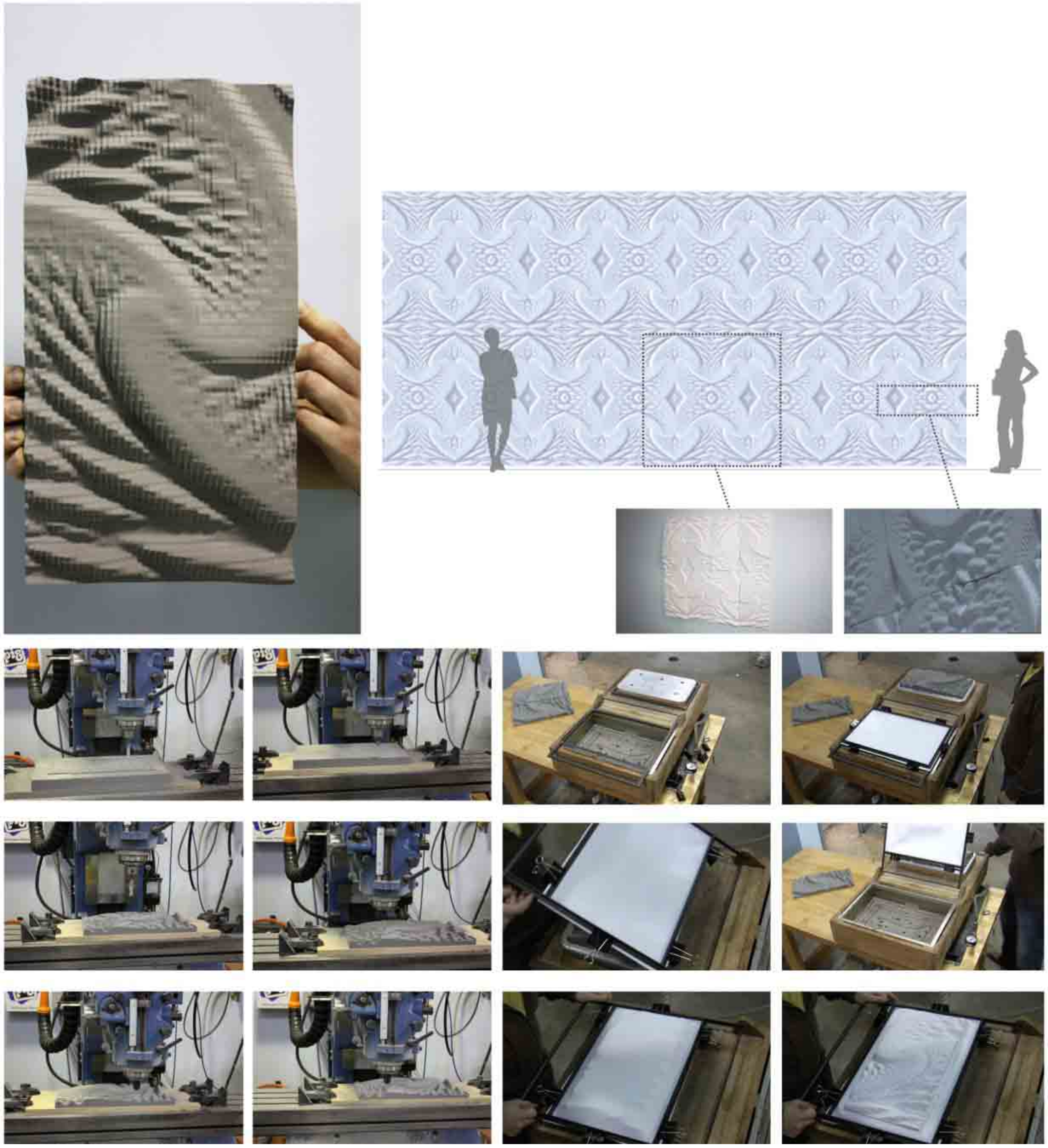


Figure 4: Fabrication process of Fractal Imprint project. The 3D voxel form was controlled by a 2D fractal image.

4.3 Smart skin

After exploring the mathematic form as building mass and skin, the method of voxelization and morphing was introduced into a studio course. The objective of the studio was to promote and assist students to experiment with a procedural network in order to design a smart skin. The studio viewed a smart skin as a hypersurface containing a large quantity of tiling components. A similar process has been used in the Suttgart city parametric skin designed by Oliver Dibrovais where the results were a glazing system that was customized based on the radiation map of its' façades   (Figure 5). This method demonstrated itself with a great power and an unlimited potential of form exploration of 3D facade.

Students were required to use the concept of voxel to develop a sequence of deformation and control nodes that tiled the building skin. Then morphing controls, either as a point attractor or a bitmap-driven input (Lemmerahl, Hovestadt, 2005), were added to the prescription and yield a matrix of morphed tiles that acted independently of each other. As a result, students created a high degree of complexity and explored the dynamic possibilities of tiling with relatively simple input information.

For each morphing voxel, or tile, students defined parameters to interactively respond to either 2D bitmap inputs or a single point attractor. By linking the 2D bitmap to each voxel's morphing weight; the building performance data such as acoustic and solar radiation analysis can be integrated into the 3D façade design. Here, the encoding of parameters as a bitmap, either generated by mathematic equation or performance analysis, let the students easily visualize the inter-connection between the data input and the corresponding variations across the 3D façade. Students followed this subjective approach and produced several unique transformable smart skin solutions.

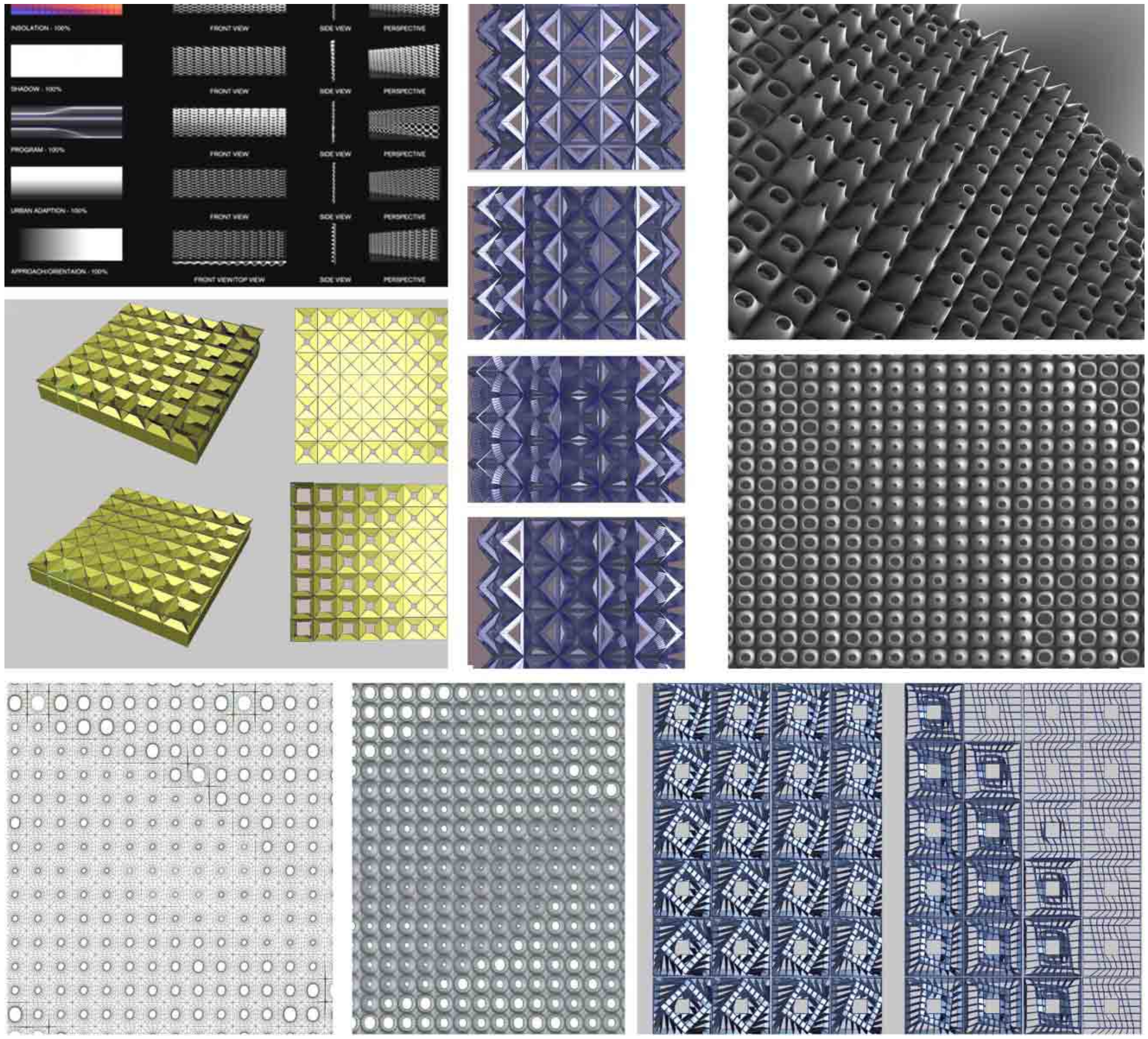


Figure 5: Top left corner. Stuttgart city parametric skin, designed by Oliver Dibrova: Bottom. Six student projects from Smart Skin studio project.

5. Conclusion

The first two research projects examined an interdisciplinary approach where computational mathematical forms were extracted as 2D contours or 3D voxels and digital fabricated in order to explore the potential as building mass and skin. The third studio project extended to the spatial interaction within the data input and each morphing voxel. The 2D bitmap driven form seeking was accomplished through the exploration of several information processing techniques to convert 2D math diagrammatic or performance data into 3D facade. The authors feel that the results were artifacts that expanded the boundary of conventional form seeking methods through mathematics.

In these experiments, the authors and students explored the manipulation of a planar surface through algorithmic equations and the molecular make-up of a surface through voxel representation. The use of mathematics generated unlimited possible artifacts that were bounded only by the parameters that a designer inputted. The adaption of several variables was used to control the repetition and resolution of these artifacts through an exhaustive combination of values. Here, existing math libraries and precedents such as Möbius and Klein bottle were used as showcases to inspire students to investigate unconventional mathematical forms in architectural design.

In terms of 3D morphology, these math-driven processes were considered as psychological change rather than just another form seeking method. “it exhibits some unexpected characteristics that we have not seen before, in terms of its form ” (Kalay, 2004). To facilitate this new design and thinking process, the marriage between the mathematics and building forms needs to have a seamless transition that allows designers not to have to learn the mathematic computing, but utilize the tools to realize the digital. Mathematical computation and “parametric relational constructions have the capacity to become more inclusive, more adaptable, less absolutist… allowing for a new model that is not built upon the persistent dialectical constructions of form/function, but more inclusive more adaptable more socially relevant providing a provisional utopia, one that is here and now”(Meredith, 2009). The authors believe this research is embarking on a practical way to overcome the tectonic constraints of mathematic forms. Furthermore, they see this as a platform that can be integrated as a procedure to seek form and then realize the digital.

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Appendix

[1] For instance, the equation of triply periodic minimal surface
sin(2*x) * cos(y) * sin(z)
+ sin(2*y) * cos(z) * sin(x)
+ sin(2*z) * cos(x) * sin(y) - 0.06 + cos(2*x) * sin(y) * cos(z)
+ cos(2*y) * sin(z) * cos(x)
+ cos(2*z) * sin(x) * cos(y)